Problem 27.15

You are given the copper wire's mass and resistance. This one of those "take the relationships we know are true and manipulate the daylights out of them!" problems. To that end, then, to determine length and cross-sectional area.

If we take the mass density to be ρ_m in kg/cubit-meters, we can write:

$$\begin{aligned} m &= \left(\rho_m \text{ kg/m}^3\right)\left(V \text{ m}^3\right) \\ &= \left(\rho_m \text{ kg/m}^3\right)\left[\left(A \text{ m}^2\right)\left(L \text{ m}\right)\right] \\ \Rightarrow \quad A &= \frac{m}{\rho_m L} \end{aligned}$$

Taking resistivity to be ρ (yes, an inconvenient symbol, but that's its symbol), we can write the resistance as:

$$\begin{aligned} R &= \rho\frac{L}{A} = \rho\frac{L}{\left(m/\rho_m L\right)} \\ &= \rho\frac{L}{\left(m/\rho_m L\right)} = \frac{\rho\rho_m L^2}{m} \end{aligned}$$

a.) the length?

Continuing:

$$\begin{aligned} R &= \frac{\rho\rho_m L^2}{m} \\ \Rightarrow \quad L &= \left(\frac{Rm}{\rho\rho_m}\right)^{1/2} \\ \Rightarrow \quad L &= \left(\frac{\left(.5\text{meters}\right)\left(10^{-3}\text{kg}\right)}{\left(1.7\text{x}10^{-8}\right)\left(8.92\text{x}10^{3}\right)}\right)^{1/2} \\ &= 1.82 \text{ m} \end{aligned}$$

b.) The wire radius? Going back to the relationship between the mass, mass density and volume, we can write:

$$\begin{aligned} m = \rho_m V \quad &\Rightarrow \quad V = \frac{m}{\rho_m} \\ \Rightarrow \quad &\left(\pi r^2 L\right) = \frac{m}{\rho_m} \end{aligned}$$

Continuing:

$$\begin{aligned} \left(\pi r^2 L\right) &= \frac{m}{\rho_m} \\ \Rightarrow \quad r &= \left(\frac{m}{\pi\rho_m L}\right)^{1/2} \\ &= \left(\frac{\left(10^{-3}\right)}{\pi\left(8.92\text{x}10^{3}\right)\left(1.82\right)}\right)^{1/2} \end{aligned}$$

$= 1.4\text{x}10^{-4}$ m with a diameter twice that value